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ORBITAL MANEUVERS FOR THE CHINA-BRAZIL SATELLITE USING CONTINUOS THRUST

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Abstract. The objective of this paper is to study the orbital maneuvers that will have to be performed by the CBERS (China-Brazil Earth Resources Satellite) satellite. The CBERS is a remote sensing satellite that is under development by China and Brazil. This satellite is planned to stay in a polar frozen orbit and orbital maneuvers will have to be performed to achieve and keep its orbital elements in a specified range. This paper will be devoted to study the maneuvers required by this satellite. After a search in the literature and analysis of the results available, one selects a scheme of solution to the problem, where a hybrid approach is used and the determination of the initial values of the Lagrange multipliers (to solve the equations given by the first order necessary conditions for a local minimum) is transformed in a direct search problem. The numerical solution of the problem in each iteration is reduced to one of nonlinear programming, which is then solved with the gradient projection method. The spacecraft is supposed to be in Keplerian motion controlled by the thrusts, that are assumed to be of fixed magnitude (either low or high) and operating in an on-off mode. Results of simulations are presented for several case studies.

Key Words: Astrodynamics, Space Maneuvers, Optimal Transfer, Optimal Control.

1. INTRODUCTION

The objective of this paper is to study the orbital maneuvers that will have to be performed by the CBERS (China-Brazil Earth Resources Satellite) satellite. In particular, the correction of the semi-major axis, eccentricity and argument of periapse, that are three of the Keplerian elements that will change during the mission due to the orbit perturbations, will be studied in detail (Carrara, 1988; Carrara and Souza, 1988).

In order to accomplish this goal, from the analyses of the alternatives of solutions available (Prado, 1989; Prado & Rios-Neto, 1993), results of the implementation and tests of one method selected to solve the problem of sending a vehicle from one orbit to another with minimum fuel expenditure are shown. The method can be used either for large orbit transfer (as a geosynchronous satellite launched by the Space Shuttle in a low parking orbit) or for small orbit correction (as the maneuvers required for station-keeping of a space station or a remote sensing satellite). The objective is to find the best way (in terms of minimum fuel expenditure) to accomplish the maneuvers required by the CBERS Satellite.

One of the first solutions of this problem was obtained by Hohmann (1925), using an impulsive approximation, the so called "Hohmann Transfer". There are many solutions proposed with this type of approximation, like the "Bi-Elliptical Transfer" (Hoelker & Silber, 1959) and the "Parabolic Transfer". Some other results using this model can be found in Prado (1993) and Broucke & Prado (1993). Later, a great attention has been given to the more realistic approach, where the thrust is considered finite. Many researchers proposed solutions for this case, as Tsien (1953), Lawden (1955), Biggs (1978; 1979), Ceballos & Rios-Neto (1981), Rios-Neto & Bambace (1981).

From the analysis of the alternatives available (Prado, 1989), one choice was made, and the optimal control (hybrid approach) was explored to develop procedures valid for high or low thrust and for large or small transfers.

Numerical results obtained in the simulations of the orbit transfer phase of the first CBERS are presented.

2. DEFINITION OF THE PROBLEM

The basic problem discussed in this paper is the problem of orbit transfer maneuvers. The objective of this problem is to modify the orbit of a given spacecraft. In the case considered in this paper, an initial and a final orbit around the Earth is completely specified. The problem is to find how to transfer the spacecraft between those two orbits in a such way that the fuel consumed is minimum. There is no time restriction involved here and the spacecraft can leave and arrive at any point in the given initial and final orbits. The maneuver is performed with the use of an engine that is able to deliver a thrust with constant magnitude and variable direction. The mechanism, time and fuel consumption to change the direction of the thrust is not considered in this paper.

The spacecraft is supposed to be in Keplerian motion controlled only by the thrusts, whenever they are active. This means that there are two types of motion:

i) A Keplerian orbit, that is an orbit obtained by assuming that the Earth's gravity (assumed to be a point of mass) is the only force acting on the spacecraft. This motion occurs when the thrusts are not firing;

ii) The motion governed by two forces: the Earth's gravity field (also assumed to be a point of mass) and the force delivered by the thrusts. This motion occurs during the time the thrusts are firing.

Figure 1 shows this situation. F_E is the gravitational force of the Earth (assumed to be a point of mass) and F_t is the force given by the thrusts.

The thrusts are assumed to have the following characteristics:

i) Fixed magnitude: The force generated by them is always of constant magnitude during the maneuver. The value of this constant is a free parameter (an input for the algorithm developed here) that can be high or low;

ii) Constant Ejection Velocity: Meaning that the velocity of the gases ejected from the thrusts is constant. The importance of this fact can be better understood by examining Prado (1989);

iii) Free angular motion: This means that the direction of the force given by the thrusts can be modified during the transfer. This direction can be specified by the angles *A* and *B*, called pitch (the angle between the direction of the thrust and the perpendicular to the line Earth-spacecraft) and yaw (the angle with the orbital plane). The motion of those angles are free;

iv) Operation in on-off mode: It means that intermediate states are not allowed. The thrusts are either at zero or maximum level all the time.

The solution is given in terms of the time-histories of the thrusts (pitch and yaw angles) and fuel consumed. Any number of "thrusting arcs" (arcs with the thrusts active) can be used for each maneuver.

Instead of time, the "range angle" (the angle between the radius vector of the spacecraft and an arbitrary reference line in the orbital plane) is used as the independent variable.

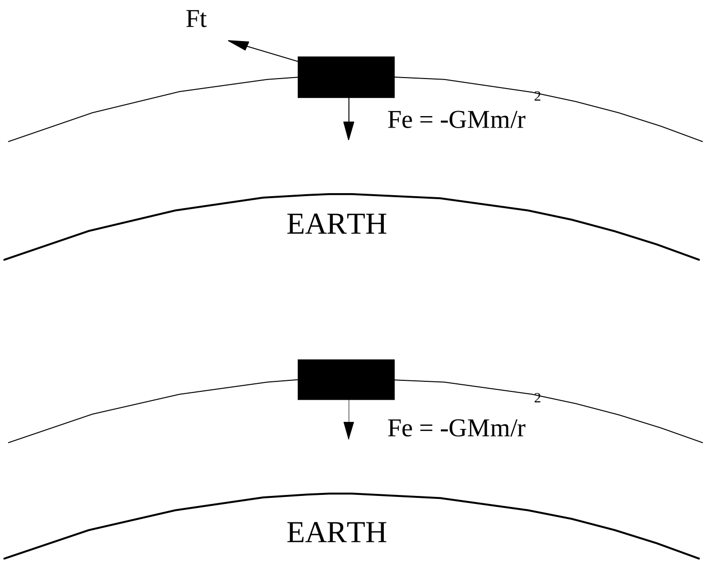

Figura 1 - Forces acting in the satellite

3. FORMULATION OF THE OPTIMAL CONTROL PROBLEM

This is a typical optimal control problem, and it is formulated as follows:

Objective Function: M_f,

where M_f is the final mass of the vehicle and it has to be maximized with respect to the control $\boldsymbol{u}(.)$, where $\boldsymbol{u}(.)$ is any continuous function;

Subject to: Equations of motion, constraints in the state (initial and final orbit) and control (limits in the angles of "pitch" and "yaw", forbidden region of thrusting and others);

And given: All parameters (gravitational force field, initial values of the satellite and others).

This approach is based on Optimal Control Theory (Bryson & Ho, 1975). First order necessary conditions for a local minimum are used. Those equations can give us the following information:

a) One set of differential equations for the Lagrange multipliers. They are called "adjoint equations". Together with the equations of motion they complete the set of differential equations to be integrated numerically at each step;

b) The "Transversality Conditions", that are the conditions to be satisfied by the Lagrange multipliers at the final time. Together with the constraints of the transfer (start in a point that belongs to the initial orbit and finishes in a point that belongs to the final orbit) these end conditions complete the set of boundary conditions to be satisfied. This problem is known as the "Two Point Boundary Value Problem" (TPBVP), because there are boundary conditions to be satisfied at the beginning and at the end of the interval of integration;

c) Maximum Principle of Pontryagin. This principle says that the magnitude of the scalar product of the Lagrange multiplier by the right-hand side of the equations of motion has to be a maximum. Working out the algebra involved we will end up with a condition for the angles of "pitch" and "yaw" that can be solved to give us their numerical values at each time.

Then, the problem becomes a problem of non-linear programming with finite dimension. This problem is then solved using the following algorithm:

i) Choose an estimate for the initial and final "range angle" (the variable that replaces the time as the independent variable) and for the initial values of the Lagrange multipliers;

ii) Integrate the adjoint equations and the equations of motion simultaneously, obtaining the instantaneous values of the "pitch" and "yaw" angles from the Maximum Principle of Pontryagin;

iii) At the end of the maneuver, verify if the boundary conditions are satisfied. If they are not satisfied update the initial values following the procedure described in the next session and go back to step i. If the constraints are satisfied the procedure is finished.

This treatment is called hybrid approach (Biggs, 1979) because it uses direct searching methods for minimization together with first order necessary conditions for a local minimum. With this approach, the problem is reduced to parametric optimization.

The main difficulty involved in this method is to find good first initial guesses for the Lagrange multipliers, because they are quantities with no physical meaning. This problem can be solved by using the method proposed by Biggs (1979). He proposes a transformation called "adjoint-control", where one guess control angles and its rates at the beginning of thrusting instead of the initial values of the Lagrange multipliers. A set of equations is developed that allow to obtain the Lagrange multipliers from the values of the initial angles of "pitch" and "yaw" and its rates. More details are available in Prado (1989) and Biggs (1979). By performing this transformation it is easier to find a good initial guess, and the convergence is faster. This hybrid approach has the advantage that, since the Lagrange multipliers remain constant during the "ballistic arcs" (arcs that have the thrusts inactive), it is necessary to guess values of the control angles and its rates only for the first "burning arc". This transformation reduces the number of variables to be optimized and, in consequence, the time of convergence.

4. NUMERICAL METHOD

To solve the nonlinear programming problem, the gradient projection method was used (Bazarra & Sheetty, 1979; Luemberger, 1973).

It means that at the end of the numerical integration, in each iteration, two steps are taken:

i) Force the system to satisfy the constraints by updating the control function according to:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i - \nabla \mathbf{f}^{\mathbf{T}} . \left[\nabla \mathbf{f} . \nabla \mathbf{f}^{\mathbf{T}} \right]^{-1} \mathbf{f} \tag{1}$$

where f is the vector formed by the active constraints;

ii) After the constraints are satisfied, try to minimize the fuel consumed. This is done by making a step given by:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i + \alpha \frac{\mathbf{d}}{|\mathbf{d}|} \tag{2}$$

where:

$$\alpha = \gamma \frac{J(\boldsymbol{u})}{\nabla J(\boldsymbol{u}) . \mathbf{d}} \tag{3}$$

$$\mathbf{d} = -\left(\mathbf{I} - \nabla\mathbf{f}^{T}\left[\nabla\mathbf{f}.\nabla\mathbf{f}^{T}\right]^{-1}\mathbf{f}\right)\nabla J(\boldsymbol{u}) \quad (4)$$

where **I** is the identity matrix, **d** is the search direction, J is the function to be minimized (fuel consumed) and γ is a parameter determined by a trial and error technique. The possible singularities in equations (1) to (4) are avoided by choosing the error margins for tolerance in convergence large enough. This procedure continues until $\left|\boldsymbol{u}_{i+1} - \boldsymbol{u}_{i}\right| < \varepsilon$ in both equations (1) and (2), where ε is a specified tolerance.

5. THE MANEUVERS REQUIRED FOR EACH ORBITAL ELEMENT

In this section, the maneuvers required by the CBERS satellite are studied for each keplerian element. It means that, for each maneuver performed here, two of the three keplerian elements that we need to change are left free and only one of them is changed to satisfy the constraints. The goal is to verify the cost of correcting each of the keplerian element to compare with the cost of correcting two or three of them. The nominal orbit for the CBERS satellite is: Semi-major axis (a) = 7148865 m, Eccentricity (e) = 0.0011, Argument of periapse (ω) = 90°

For the purposes of this first study, it is assumed that the maneuver will be performed when the keplerian elements have the values given in Table 1.

Table 1 – Keplerian elements before the manneuver

Keplerian element	Value
Semi-major axis	7122237 m
Eccentricity	0.0014161
Argument of Periapse	98.69375°

1) Change in semi-major axis

For this maneuver it is assumed that the semi-major axis has the value given by the limit shown in Table 1, while the eccentricity and the argument of periapse are left free of the control. The solution obtained for this case is shown in Figure 2. The "pitch" angle is zero for all the maneuvers shown here because this is a planar maneuver. Two burning arcs were allowed for each maneuver, unless explicit mentioned in the text.

The final orbit obtained is: semi-major axis = 7148.865 km, eccentricity = 0.0017, inclination = 10.00 degrees, longitude of the ascending node = 20.00 degrees, argument of the perigee = 102.90 degrees, true anomaly = 106.69 degrees. The fuel consumption is 4.9132 kg and the burning time is 17.4 minutes.

2) Change in eccentricity

For this maneuver it is assumed that the eccentricity has the value given by the limit shown in Table 1, while the semi-major axis and the argument of periapse are left free of the control.

The solution obtained for this case is shown in Figure 3.

The final orbit obtained is: semi-major axis = 7126.310 km, eccentricity = 0.0011, inclination = 10.00 degrees, longitude of the ascending node = 20.00 degrees, argument of the perigee = 96.02 degrees, true anomaly = 103.48 degrees. The fuel consumption is 1.2676 kg and the burning time is 4.5 minutes.

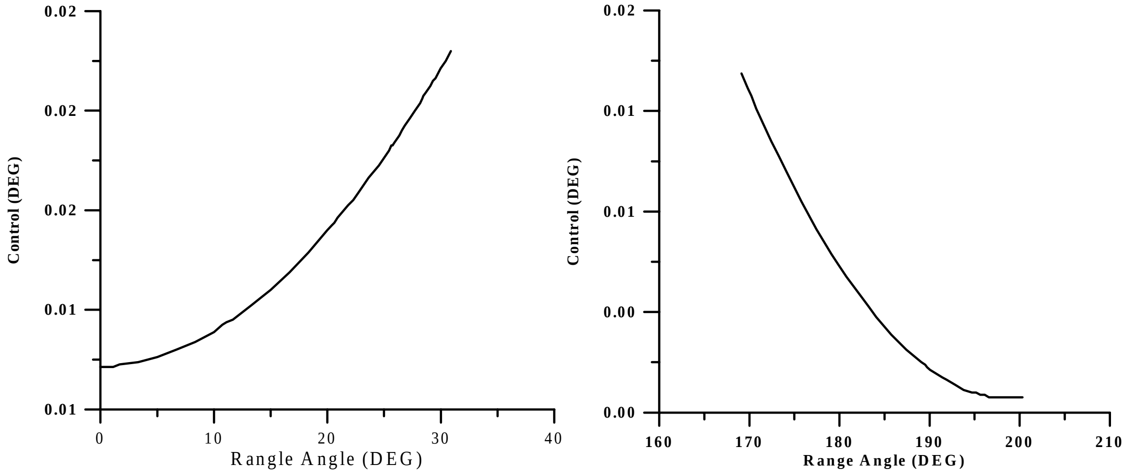

Fig. 2 – Control to be used for the maneuver in semi-major axis.

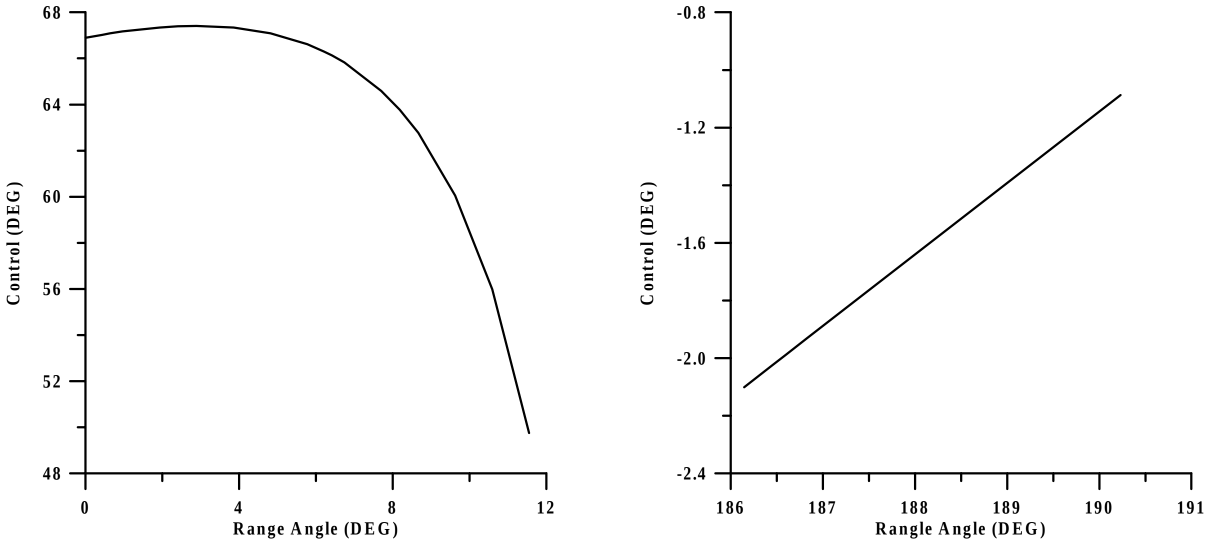

Fig. 3 – Control to be used for the maneuver in eccentricity.

3) Change in argument of periapse

For this maneuver it is assumed that the argument of periapse has the value given by the limit shown in Table 1, while the semi-major axis and the eccentricity are left free of the control. The solution obtained for this case is shown in Figure 4.

The final orbit obtained is: semi-major axis = 7130.150 km, eccentricity = 0.0015, inclination = 10.00 degrees, longitude of the ascending node = 20.00 degrees, argument of the perigee = 90.00 degrees, true anomaly = 107.45 degrees. The fuel consumption is 1.4650 kg and the burning time is 5.2 minutes.

6. THE MANEUVERS FOR EACH TWO OF THE KEPLERIAN ELEMENTS

In this section, the maneuvers required by the CBERS satellite are studied to change each two of the keplerian elements. It means that, for each maneuver performed here, one of the three keplerian elements that we need to change is left free of the control, while the two other are changed to reach the required values.

1) Change in semi-major axis and eccentricity

For this maneuver it is assumed that the semi-major axis and the eccentricity have the values given by the limit shown in Table 1, while the argument of periapse is left free of the control. The solution obtained for this case is shown in Figure 5.

The final orbit obtained is: semi-major axis = 7148.865 km, eccentricity = 0.0011, inclination = 10.00 degrees, longitude of the ascending node = 20.00 degrees, argument of the perigee = 116.57 degrees, true anomaly = 99.14 degrees. The fuel consumption is 5.0867 kg and the burning time is 18.0 minutes.

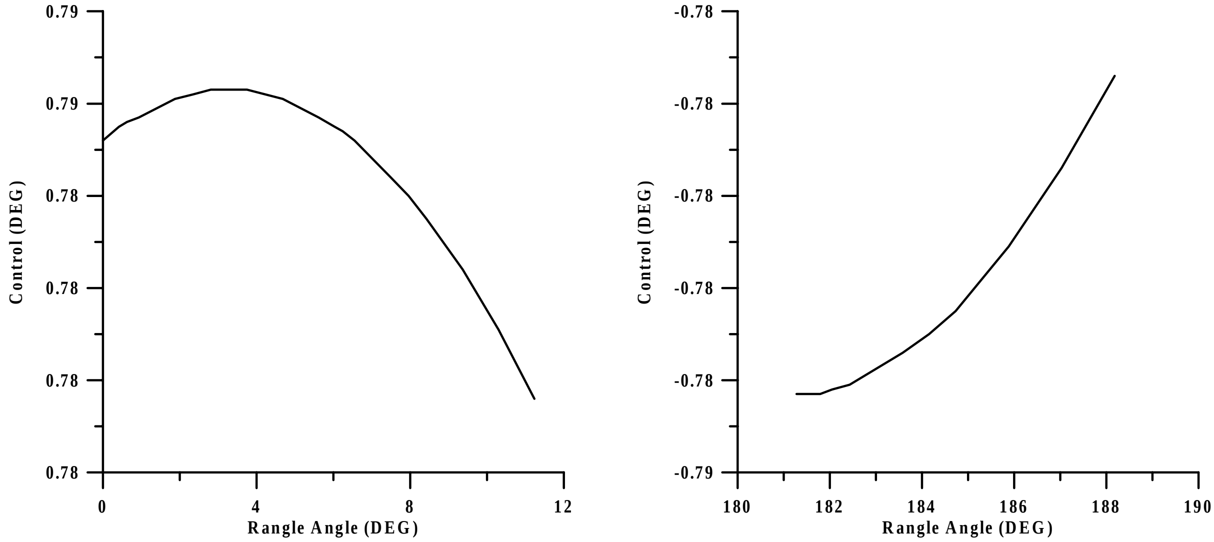

Fig. 4 – Control to be used for the maneuver in argument of periapse.

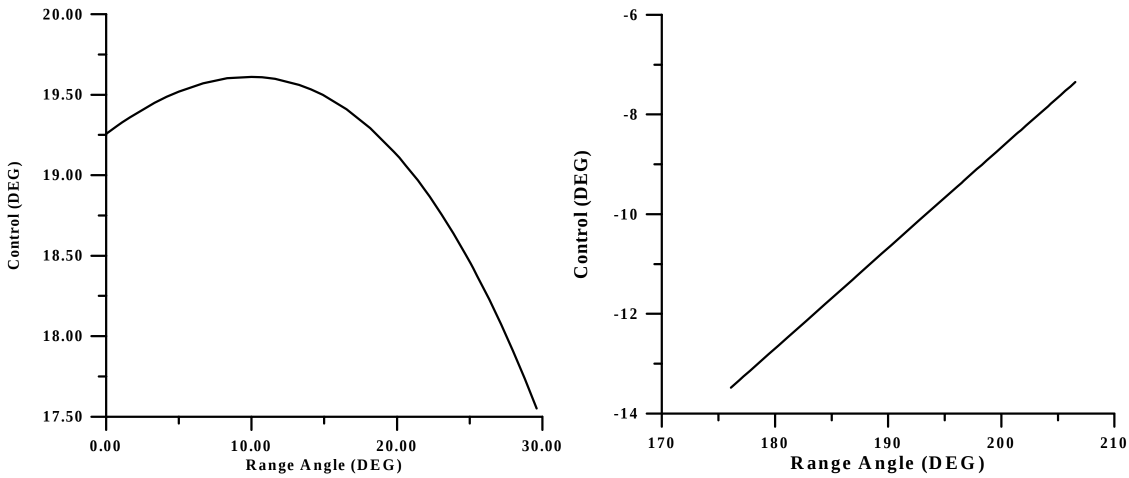

Fig. 5 – Control to be used for the maneuver in semi-major axis and eccentricity.

2) Change in semi-major axis and argument of periapse

For this maneuver it is assumed that the semi-major axis and the argument of periapse have the values given by the limit shown in Table 1, while the eccentricity is left free of the control. The solution obtained for this case is shown in Figure 6.

The final orbit obtained is: semi-major axis = 7148.865 km, eccentricity = 0.0019, inclination = 10.00 degrees, longitude of the ascending node = 20.00 degrees, argument of the perigee = 90.00 degrees, true anomaly = 117.82 degrees. The fuel consumption is 4.9138 kg and the burning time is 17.4 minutes.

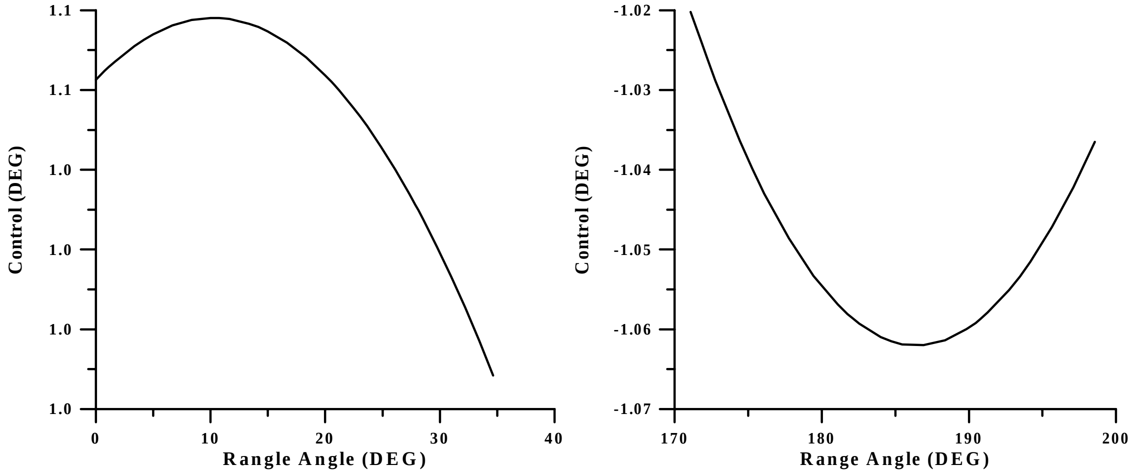

Fig. 6 – Control to be used for the maneuver in semi-major axis and argument of periapse.

3) Change in argument of periapse and eccentricity

For this maneuver it is assumed that the eccentricity and the argument of periapse have the values given by the limit shown in Table 1, while the semi-major axis is left free of the control. The solution obtained for this case is shown in Figure 7.

The final orbit obtained is: semi-major axis = 7130.721 km, eccentricity = 0.0011, inclination = 10.00 degrees, longitude of the ascending node = 20.00 degrees, argument of the perigee = 90.00 degrees, true anomaly = 110.34 degrees. The fuel consumption is 1.9494 kg and the burning time is 6.9 minutes.

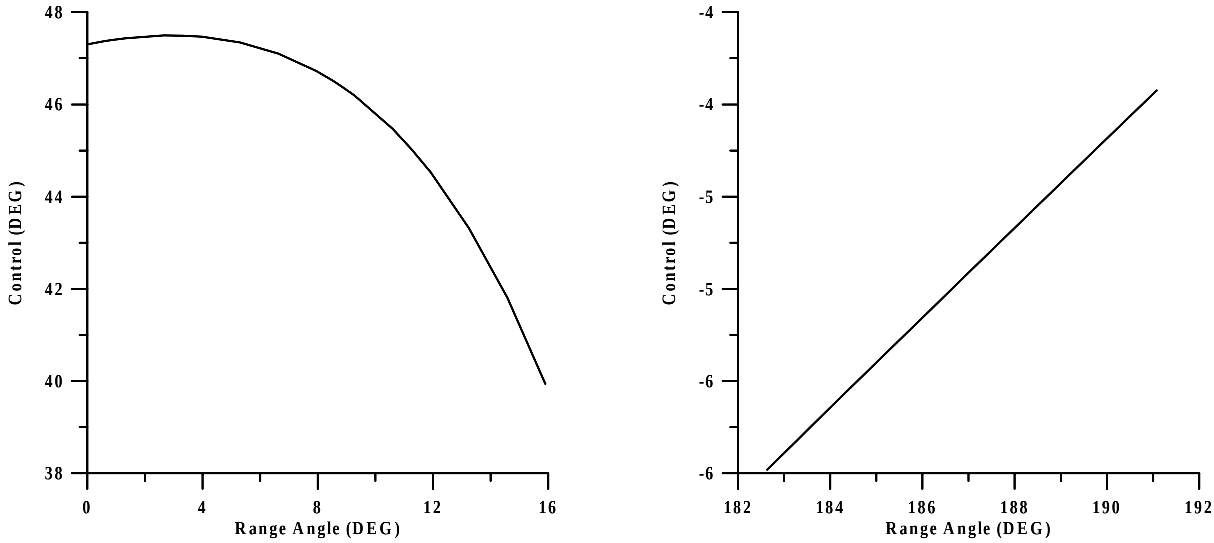

Fig. 7 – Control to be used for the maneuver in eccentricity and argument of periapse.

7. THE COMPLETE MANEUVER REQUIRED BY THE CBERS SATELLITE

In this section, the maneuvers required by the CBERS satellite are studied for all the keplerian elements at the same time. It means that all the three keplerian elements are supposed to be in the limit shown in Table 1 and all of them will be changed . The solution obtained for this case is shown in Figure 8.

The final orbit obtained is: semi-major axis = 7148.865 km, eccentricity = 0.0011, inclination = 10.00 degrees, longitude of the ascending node = 20.00 degrees, argument of the perigee = 90.00 degrees, true anomaly = 125.81 degrees. The fuel consumption is 5.1564 kg and the burning time is 18.3 minutes.

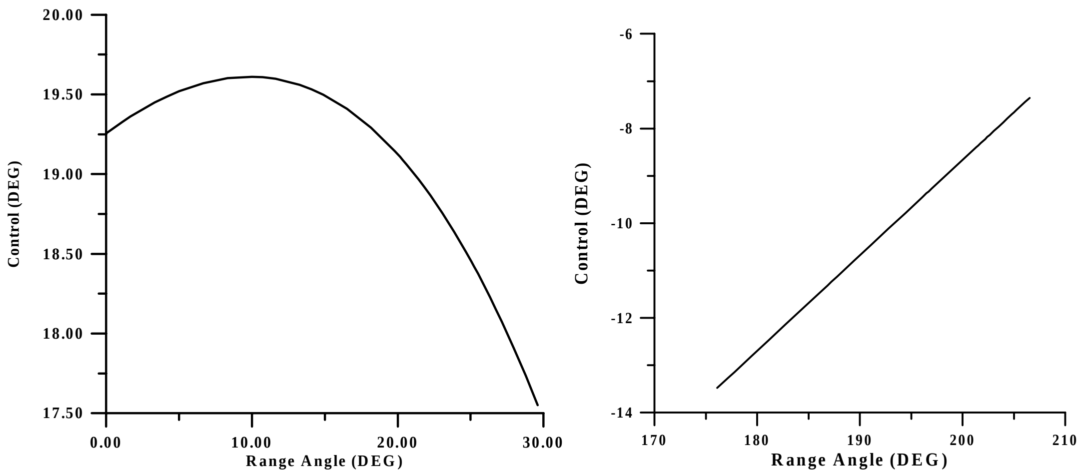

Fig. 8 – Control to be used for the complete maneuver.

Table 1 shows the consumption obtained for all the the simulations shown here.

Table 1 - Fuel consumed for all the maneuvers simulated

Maneuver	Consumption (kg)	Burning Time (min)
Semi-major axis	4.9132	17.4
Eccentricity	1.2676	4.5
Argument of Periapse	1.4650	5.2
Semi-major axis and Eccentricity	5.0867	18.0
Semi-major axis and Arg. Periapse	4.9138	17.4
Eccentricity and Arg. Periapse	1.9494	6.9
Complete (2 arcs)	5.1564	18.3
Complete (4 arcs)	4.9182	17.5

10 - CONCLUSIONS

Optimal control was explored to generate algorithms to obtain solutions for the minimum fuel maneuvers required by the first CBERS satellite.

From the simulations shown in this paper it is possible to obtain several conclusions. First of all, it is easy to see that, considering each keplerian element individually, the cost to correct semi-major axis is a lot larger than correcting eccentricity or argument of periapse. After that, the argument of periapse has a cost a little bit larger then the eccentricity, in order to be corrected.

When two keplerian elements are corrected in the same maneuver, the cost is smaller than the total cost required for the corrections of the two elements. Usually, the cost is a little bit larger than the cost required by the most expensive maneuver, which means that the additional cost to correct two instead of one orbital element is small.

For the complete maneuver, we verified that all the keplerian elements can be corrected with a cost a little bit larger than the cost to correct the semi-major axis only (the most expensive single correction). After that, a complete maneuver with four burning arcs was simulated. The consumption obtained was 4.9182 kg, that is smaller than the fuel required by the maneuver with two burning arcs. This is expected, because with more burning arcs there are more parameters to be otimized and it allow us to reduce the total cost. A maneuver with eight burning arcs was also simulated, but the fuel consumed did not decrease by a significant ammount.

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